

4.9 Problems

Problem 1. Use Composite Simpson's rule and the given value of n to approximate the following improper integrals:

1. $\int_0^1 x^{-1/4} \sin(x) dx, n = 4$

2. $\int_0^1 \frac{e^{2x}}{x^{2/5}} dx, n = 6$

Problem 2. Use the transformation $t = x^{-1}$ and the composite Simpson's rule for $n = 4$ to compute:

$$\int_1^{\infty} \frac{1}{x^2 + 9} dx$$

5.1 Problems

Problem 3. Use Theorem 5.4 to show that the following initial-value problems have a unique solution, and find the solution:

1. $y' = y \cos(t), 0 \leq t \leq 1, y(0) = 1$

2. $y' = -\frac{2}{t}y + t^2 e^t, 1 \leq t \leq 2, y(1) = \sqrt{2}e$

Problem 4. Show that the given equation implicitly defines a solution. Approximate $y(2)$ using Newton's method:

$$y' = -\frac{y^3 + y}{(3y^2 + 1)t}$$

for $1 \leq t \leq 2, y(1) = 1. y^3 t + yt = 2$